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SIMPLE MATHEMATICAL MODELS OF THE EXPLOSIVE FAILURE OF A SPACECRAFT*

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Here, we propose simple mathematical models that make it possible to calculate the number of fragments formed in the failure of a spacecraft as a result of an internal explosion. The models also determine the initial velocity of the fragments at the moment of failure.

1. Models of Failure. We make the following simplifying assumptions.

1. The hull of the spacecraft is modeled by a cylindrical or spherical shell.
2. The shell is thin: $h/r \ll 1$ (where h is the thickness and r is the radius of the shell).
3. The effect of the explosion is modeled by the pressure $p = p(t)$, which is dependent on time t and is uniformly distributed over the inside surface of the shell. The characteristic time of action of the load $\tau \gg h/a_0$ (where a_0 is sonic velocity in the shell material).
4. The material of the shell is elastoviscoplastic and the deformation process is adiabatic.
5. The limiting unit dissipation [1, 2] is taken as the criterion of the beginning of failure.
6. It is assumed that fracture of the shell occurs due to the expenditure of elastic energy accumulated in the shell by the moment $t = t_*$ at which failure begins; the work of the external forces during fracture is ignored; cleavage fracture is not considered [3, 4].

By virtue of the first three assumptions, the problem of deformation of a thin shell can be regarded in a first approximation as a homogeneous cylindrical or spherical problem.

Then the equation of motion has the form

$$\rho\dot{v} = \frac{p(t)}{h} - \alpha\frac{\sigma_\theta}{r}, \tag{1.1}$$

where ρ is the running density; v is radial velocity; r is the running value of shell radius; σ_θ is the circumferential force (the stress averaged over the thickness of the shell); the dot denotes a substantial derivative with respect to time; in (1.1) and below, $\alpha = 1$ corresponds to the cylindrical case, $\alpha = 2$ corresponds to the spherical case.

The rate of circumferential deformation is determined as

$$\dot{\varepsilon}_\theta = \frac{v}{r},$$

while other strains are absent due to the thinness of the shell.

The equation of the mass conservation law has the form $\dot{\rho}/\rho = -\alpha\dot{\varepsilon}_\theta$. It follows from this equation that

$$\rho = \rho_0 \exp(-\alpha\varepsilon_\theta).$$

We take the equations of state of the elastoviscoplastic material in the Pezhin form [5]; with allowance that in the cylindrical case

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$$\sigma_\theta = \sigma + S_\theta, \quad \sigma_z = \sigma + S_z, \quad \sigma_r = \sigma + S_r = 0, \quad S_\theta + S_z + S_r = 0,$$

while in the spherical case

$$\sigma_\theta = \sigma + S_\theta, \quad \sigma_\varphi = \sigma_\theta, \quad \sigma_r = \sigma + S_r = 0, \quad 2S_\theta + S_r = 0,$$

these equations reduce to the form

$$\dot{S}_\theta = \frac{1}{\alpha}\frac{4}{3}\mu\dot{\varepsilon}_\theta - \frac{\mu}{\eta} S_\theta \frac{|S_\theta| - \frac{1}{\alpha}\frac{2}{3}J_0}{|S_\theta|} H(|S_\theta| - \frac{1}{\alpha}\frac{2}{3}J_0), \quad \sigma_\theta = \alpha\frac{3}{2}S_\theta.$$

Here, μ is the shear modulus; η is the absolute viscosity of the material; J_0 is the static elastic limit in simple tension; H(x) is the Heaviside unit function. Here, the stress tensor σ_{ij} is broken up into spherical $\sigma\delta_{ij}$ and deviatoric S_{ij} parts ($\sigma_{ij} = \sigma\delta_{ij} + S_{ij}$) and we assume that the strain rates can also be divided into elastic and plastic rates; the plastic flow is incompressible:

$$\dot{\varepsilon}_\theta = \dot{\varepsilon}_\theta^e + \dot{\varepsilon}_\theta^p, \quad \dot{\varepsilon}_r = \dot{\varepsilon}_r^e + \dot{\varepsilon}_r^p = 0, \quad \dot{\varepsilon}_z = \dot{\varepsilon}_z^e + \dot{\varepsilon}_z^p = 0, \quad \dot{\varepsilon}_\theta^p + \dot{\varepsilon}_z^p + \dot{\varepsilon}_r^p = 0$$

in the cylindrical case and

$$\dot{\varepsilon}_\theta = \dot{\varepsilon}_\theta^e + \dot{\varepsilon}_\theta^p, \quad \dot{\varepsilon}_\varphi = \dot{\varepsilon}_\theta, \quad \dot{\varepsilon}_r = \dot{\varepsilon}_r^e + \dot{\varepsilon}_r^p = 0, \quad 2\dot{\varepsilon}_\theta^p + \dot{\varepsilon}_r^p = 0$$

in the spherical case.

The unit (per unit mass) elastic energy E and mechanical dissipation D are calculated from the formulas

$$E = \alpha \int_0^t \frac{\sigma_\theta}{\rho}\dot{\varepsilon}_\theta^e\,dt, \quad D = \alpha \int_0^t \frac{\sigma_\theta}{\rho}\dot{\varepsilon}_\theta^p\,dt.$$

Specific internal energy U = E + D.

The rates of elastic and plastic deformation are calculated from the formulas

$$\dot{\varepsilon}_\theta^e = \frac{\dot{S}_\theta}{2\mu} + \alpha\frac{\dot{\varepsilon}_\theta}{3}, \quad \dot{\varepsilon}_\theta^p = \dot{\varepsilon}_\theta - \dot{\varepsilon}_\theta^e.$$

The limiting unit dissipation is the criterion of the beginning of failure of the shell. For the chosen model of the medium, this quantity reduces to the mechanical dissipation:

$$D = \alpha \int_0^{t_*} \frac{\sigma_\theta}{\rho}\dot{\varepsilon}_\theta^p\,dt = D_*$$

(D_* is the constant of limiting unit dissipation, determined from experiments involving the clevage fracture of plates in an in-plane collision [1, 2]).

The number of fragments expected to be formed by explosion of the shell $\langle N \rangle$ is found from the balance of elastic strain energy and the work done in the rupture of the material. For a cylindrical shell, the balance equation has the form

$$\rho_0(\pi d_0 h)E_* = \gamma h\langle N \rangle,$$

where γ is the specific energy expended on the formation of a unit of free surface; E_* is the density of stored energy at the moment $t = t_*$; d_0 is the initial diameter of the shell. Then for the cylindrical shell

$$N = \left[\pi\frac{\rho_0 d_0 E_*}{\gamma}\right] \tag{1.2}$$

(the bracket denotes the integral part of the number).

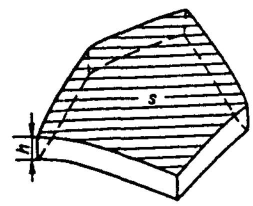

Fig. 1

We must make one more simplifying assumption to obtain the formula for the number of fragments of the spherical shell. We assume that all of the fragments are of the same size (the characteristic area of the outside surface of a fragment is s, Fig. 1) and that the ratio of s to the square of the half-perimeter p of the contour bounding this surface is constant:

$$k = \frac{s}{p^2}.$$

We then take 0.2 as the value of the form factor k (k = 0.25 for a square, k = $\sqrt{3}/9$ for a regular triangle, and k = $1/\pi$ for a circle). We can also discard the assumption that all of the fragments are of the same size and assume that the masses (and, thus, the areas s) of the fragments are distributed in accordance with a certain law. Here, we take the Rosing—Rammler law, which is widely used and is a special case of general probability representations [6]:

$$M(q) = M\left\{1 - \exp\left(-\left(\frac{q}{q_0}\right)^n\right)\right\}.$$

Here, M(q) is the total mass of the fragments having a mass less than q; q_0 is the characteristic mass; n is an exponent ensuring the uniformity of the fragmentation. The entire failure spectrum can be predicted in the given case, but we will limit ourselves here to solving the problem of the fragmentation of a spherical shell within the framework of the assumptions made above.

We then use the following system of equations to calculate the mean number $\langle N \rangle$ of fragments formed from the spherical shell:

$$k = \frac{s}{p^2}, \quad hp\gamma\langle N\rangle = \pi d_0^2 h \rho_0 E_*, \quad s\langle N\rangle = \pi d_0^2.$$

From here, we find the number of fragments N of the spherical shell:

$$N = \left[\pi k\left(\frac{\rho_0 d_0 E_*}{\gamma}\right)^2\right]. \tag{1.3}$$

The initial velocity of the fragments V_0 is assumed to be equal to the radial rate of expansion of the shell at the moment of fracture:

$$V_0 = v\Big|_{t=t_*}.$$

2. Results of Calculations. The main calculations were performed for duralumin shells with the diameter d_0 = 3 m and thickness h = 0.003 m: ρ_0 = 2700 kg/m^3; μ = 27.6 GPa; η = 10 kPa·sec; J_0 = 0.29 GPa; γ = 400 kJ/m^2; D_* = 30 kJ/kg. Internal pressure was calculated from the formula

$$p(t) = \frac{p_0}{(1 + t/\tau)^3},$$

where p_0 is the initial pressure on the shell; τ is the characteristic time of action of the load.

TABLE 1

p_0, GPa	τ, msec	N	l/h	U_0, m/sec	t_*, msec	$(\varepsilon_\theta^p)_*$	I_*, kPa·sec
0,01	1,00	979	3,21	439	1,640	0,207	4,28
0,01	10,00	990	3,13	1057	0,951	0,189	8,31
0,01	100,00	992	3,17	1148	0,904	0,186	8,92
0,1	0,10	980	3,21	541	0,996	0,196	4,95
0,1	1,00	1026	3,06	2539	0,277	0,145	9,40
0,1	10,00	1044	3,01	3080	0,242	0,137	23,30
1	0,01	980	3,21	547	0,899	0,194	4,95
1	1,00	1242	2,53	7481	0,063	0,085	57,70
1	100,00	1268	2,48	7903	0,061	0,083	60,80
10	0,01	1181	2,66	6096	0,043	0,079	47,70
10	0,10	2009	1,56	18203	0,018	0,048	141,00
10	1,00	2213	1,42	21114	0,017	0,046	164,00

TABLE 2

p_0, GPa	τ,msec	$N \cdot 10^{-3}$	l/h	U_0, m/sec	t_*, msec	$(\varepsilon_\theta^p)_*$	I_*, kPa·sec
0,01	10,00	1097	1,52	597	0,688	0,0402	6,23
0,01	100,00	1097	1,52	653	0,660	0,0402	6,53
0,1	1,00	1098	1,51	2021	0,205	0,0400	15,50
0,1	10,00	1100	1,51	2409	0,187	0,0401	18,20
1	0,01	1098	1,52	447	0,415	0,0402	4,95
1	10,00	1257	1,41	7632	0,057	0,0366	56,50
5	0,01	1099	1,51	3105	0,078	0,0398	21,40
5	0,10	1670	1,23	12390	0,026	0,0273	93,30
5	1,00	1919	1,15	15104	0,023	0,0252	113,00
10	0,01	1173	1,47	6145	0,040	0,0357	47,50
10	0,10	2294	1,05	18170	0,017	0,0221	137,00
10	1,00	2613	0,97	20999	0,016	0,0210	157,00

Some of the results of the calculations are shown in Tables 1 (cylindrical shell) and 2 (spherical shell) and Figs. 2 and 3 (a — cylindrical shell, b — spherical shell). We used the following notation: $(\varepsilon_\theta^p)_*$ is the plastic strain accumulated by the moment $t = t_*$; l is the characteristic dimension of a fragment ($l = \pi d_0/N$ for the cylindrical shell and $l = 2d_0\sqrt{\pi k/N}$, for the spherical shell); I_* is the impulse of the applied load:

$$I_* = \int_0^{t_*} p(t)\,dt.$$

It is evident from the calculations that the number of fragments N depends heavily on the character of the applied load $p = p(t)$: N increases with an increase in the intensity of the load p_0 and its duration τ. In the context of the formulation of the problem, this result is a consequence of the sensitivity of the material to the rate of plastic deformation. An increase in p_0 and τ is accompanied by a decrease in plastic strains accumulated in the shell by the moment when failure begins $t = t_*$.

The impulse I_* transmitted to the shell by the moment $t = t_*$ determines the number of fragments N. Conversely, the number N can be used to determine I_*. This is clear from Fig. 2, which shows the dependence of N on I_*. The theoretical points obtained with different combinations of p_0 and τ lie on single curves. A similar pattern is seen for the dependence of the initial velocity of the fragments V_0 on I_*. The same curve (Fig. 3) describes the cylindrical and spherical shells. The dependences of N on I_* and V_0 on I_* are close to linear; a nonlinear section is seen only at small I_*.

Let us compare the results of our calculations with well-known experimental and theoretical results on shell fragmentation published in the literature.

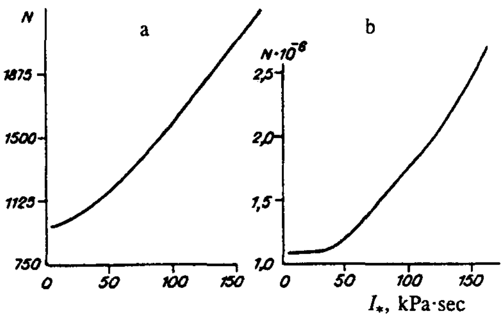

Fig. 2

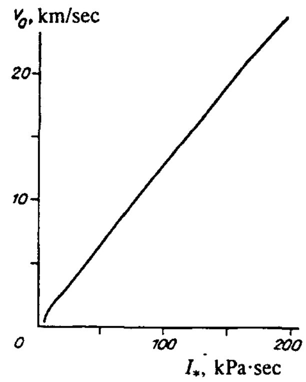

Fig. 3

Many investigations [7-14, etc.] have examined the fragmentation of cylindrical metallic (mainly steel) shells. Formulas for the mean number of fragments have been obtained from theoretical studies based on different assumptions regarding the mechanisms responsible for deformation and fracture. The formulas in [7-12] have the general form:

$$N \sim \frac{\pi d_0 V_0}{Lc}. \tag{2.1}$$

Here, L is the characteristic linear dimension of the ring (thickness, radius, etc.); c is the characteristic velocity. As was shown in [11], the formula from [8] can be represented in the form

$$N = \pi V_0 \sqrt{\frac{\rho_0}{6 J_0 \varepsilon_f}},$$

and from [9] — in the form

$$N = \frac{2\pi V_0}{v_c} \quad \left(v_c = \int_0^{\varepsilon_f} \sqrt{\frac{1}{\rho_0}\frac{d\sigma}{d\varepsilon}}\, d\varepsilon\right),$$

where ε_f is the critical strain; v_c is the critical collision velocity.

The linear dependence of the number of fragments on the radial velocity of the cylinder (2.1) has been confirmed by a number of experiments [13].

A different (quadratic) dependence of N on V_0 was obtained in [14]:

$$N = 3\pi\left(\frac{1}{2} J_0 d + \eta V_0\right)^2 (2 d_0 E_0 \gamma)^{-1}. \tag{2.2}$$

Here, E_0 is the elastic modulus; the material was assumed to be viscoplastic; $\varepsilon_f << 1$; work on fracture of the material was completed as a result of the accumulation of elastic energy.

The experiments (steel tubes, $h/d_0 = 0.021$-0.025) conducted in [14] are satisfactorily described by this formula. It follows from them that the width of the fragments is 2-8 times greater than their thickness. This finding is consistent with our results (see Table 1).

The expressions found here for the dependence of the number of fragments on V_0 (1.2), (1.3) are not as explicit as (2.1) or (2.2). However, it is evident from the calculated results that in both cases (cylindrical and spherical shells) the dependences are close to quadratic for small I_* and close to linear for large I_* (see Figs. 2 and 3, Tables 1 and 2).

The authors of [15, 16] presented results of expansion of cylindrical and spherical shells of different materials due to explosion. Various failure criteria were examined for possible use in practical calculations. It was shown that the criterion used in the present investigation [3, 4] (Proposition 6) gives good results. Unfortunately, no data were reported on the fragmentation of the shells.

The study [17] examined the fragmentation of a cylindrical shell in a formulation somewhat different from that used above, to wit: the radial force (the stress averaged over the thickness of the shell) σ_r was not negligible relative to the circumferential force σ_θ and it was assumed that $\sigma_r = -p(t)/2$ (since the pressure p(t) was assigned on the inside surface, so that $\sigma_r = -p(t)$, while the outside surface was free of loads — $\sigma_r = 0$). Here, we adopted the assumption usually made in the theory of thin shells: $\sigma_r = 0$. As a result, while the expression $\sigma_\theta = 3/2S_\theta - 1/2p(t)$, was obtained in [17] for the circumferential force, we obtained $\sigma_0 = 3/2S_\theta$. There is therefore a marked difference between the calculated number of fragments shown in Table 1 for our study and for [17], particularly when large internal pressures p(t) are assigned. The case of spherical geometry was not considered in [17].

Thus, as a first approximation, the simple models proposed here make it possible to evaluate the number of fragments and their initial velocity in the rupture of the hull of a spacecraft as a result of an internal explosion. Subsequent refinements of the model should focus first of all on the dependence of the load on time and its distribution over the surface of the craft. Next in order of importance are refining the distribution of the fragments according to mass, allowing for the additional acceleration of fragments of different masses by the explosion products after rupture of the shell, allowing for the actual geometry of the craft, more fully accounting for the actual physical processes that accompany the high-rate deformation and fracture of the structural materials, and refining the main parameters of the model.

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